Lecture 8: The New Growth Theory: Lucas Model

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Outline

- Endogenising technological progress (brief review)
- Incorporating human capital
  - An augmented Solow-Swan model
  - Lucas (1988) human capital model
  - Technology transfer with education
- Final comments on growth theory
- Review of the original Kaldor stylised facts and introducing the new Kaldor facts

Endogenising Technological Progress (Brief Review)

- In this stream of literature, sustained growth is driven by sustained growth in technology where the latter is somehow chosen by the agents in the economy.
- The main focus of this literature has been to identify what A is, and specify how it enters the production function and how it is accumulated over time.
- In particular, R&D-based growth models have gained significance in this literature, where growth is driven by technological change that results from the research and development efforts of profit-maximising agents.
- This line of research seems plausible in accounting for world wide growth (i.e. understanding technological progress seems central to understand worldwide growth).

Incorporating Human Capital

- Another stream of the new growth literature is to incorporate human capital and model its evolution over time. We will briefly introduce several examples and see whether human capital may help explain income differences across countries.
- The first is an augmented Solow-Swan model, see Mankiw, Romer and Weil (1992), where human capital is nothing but an ordinary input in the aggregate production function.
- The second is an endogenous growth model through human capital accumulation, Lucas (1988).

Solow-Swan Model with Human Capital (1 of 4)

- Production function is assumed as:

$$Y_t = K_t^{\alpha}[A_t H_t]^{1-\alpha}$$

  where H is the total amount of productive services supplied by workers. That is, it is the total contribution of workers of different skill levels to production (including contributions of both raw labour and human capital).
- The dynamics of K and A are the same as in the Solow-Swam model. A grows at an exogenous rate g. An exogenous fraction s of output is saved, and capital depreciates at an exogenous rate of δ.

Solow-Swan Model with Human Capital (2 of 4)

- The model revolves around its assumptions about how the quantity of human capital, H, is determined. The model assumes that each worker's human capital depends only on his or her years of education.

$$H_t = L_t G(x)$$

  where L_t is the number of workers in period t and grows at an exogenous rate n, and $G(x)$ is a function that defines human capital per worker as a function of years of education per worker, $G' > 0$, $G'' > 0$, x is a constant over a time.

- The dynamics of the model are exactly like those of the Solow-Swan model, if we define $k_t = \frac{K_t}{A_t L_t G(x)}$.

Solow-Swan Model with Human Capital (3 of 4)

- As in Solow-Swan model, k_t converges to a steady state,

$$k^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

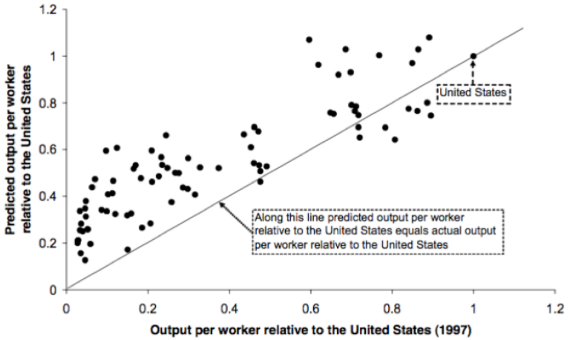
  Once k reaches k^*, the economy is on a balanced growth path with output per worker growing at rate g.
- Conditional convergence once more with human capital.
  - Output per worker in country i relative to output per worker in country j is given by (let $G(x) = e^{\eta x}$)

$$\tilde{y}_{ij} = \frac{Y_i/L_i}{Y_j/L_j} = e^{\eta(x_i - x_j)}\left(\frac{s_i}{s_j} \times \frac{n_j+g+\delta}{n_i+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

  - Notice that predicted relative GDP per capita is conditional on the saving rate, rate of production growth and average numbers of years education.

Solow-Swan Model with Human Capital (4 of 4)

- Adding human capital greatly improves the prediction of the Solow-Swan model in terms of conditional convergence.



- The figure suggests that human capital is an important explanation for differences in GDP per capita among countries, but it is also evident that there remains a considerable part of the variation that is yet to be explained.

- A representative agent model.
- Each agent has 1 unit of time that he can allocate to produce consumption goods and accumulate education (human capital).
- **Periodic utility** function is given by $u(C_t) = \frac{C_t^{1-\theta}-1}{1-\theta}$, where $\theta > 1$.
- The **production function** is given by $Y_t = K_t^{\alpha}(\phi_t H_t)^{1-\alpha}$
  - H_t denotes human capital
  - ϕ_t is the fraction of hours devoted to work
  - $1 - \phi_t$ is the fraction devoted to education
- New human capital is produced using the CRS technology.

$$H_{t+1} = B(1 - \phi_t)H_t$$

where $B > 0$ measures the return to education, and $H_0 > 0$ is given

A Model of Human Capital Accumulation: Lucas (1988) Model (2 of 4)

- As in Ramsey model, we can find the competitive equilibrium from a social planner's problem:

$$\max_{\{\phi_t, H_{t+1}, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta} - 1}{1-\theta}$$

$$\begin{aligned} s.t. \qquad C_t + K_{t+1} &= K_t^{\alpha}(\phi_t H_t)^{1-\alpha} + (1-\delta)K_t & (1) \\ H_{t+1} &= B(1-\phi_t)H_t & (2) \end{aligned}$$

where K_0 and H_0 are given

A Model of Human Capital Accumulation: Lucas (1988) Model (3 of 4)

- Set up the Lagrangian equation

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{ \frac{[K_t^{\alpha}(\phi_t H_t)^{1-\alpha} + (1-\delta)K_t - K_{t+1}]^{1-\theta} - 1}{1-\theta} + \lambda_t [B(1-\phi_t)H_t - H_{t+1}] \}$$

- The F.O.C.s are (see Appendix 1 for detail)

$$\phi_t : C_t^{-\theta} [\frac{(1-\alpha)Y_t}{\phi_t}] = \lambda_t B H_t \quad (3)$$

$$H_{t+1} : \lambda_t = \beta \{ C_{t+1}^{-\theta} [\frac{(1-\alpha)Y_{t+1}}{H_{t+1}}] + \lambda_{t+1} B(1-\phi_{t+1}) \} \quad (4)$$

$$K_{t+1} : C_t^{-\theta} = \beta \{ C_{t+1}^{-\theta} [\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)] \} \quad (5)$$

A Model of Human Capital Accumulation: Lucas (1988) Model (4 of 4)

- Solving Eq.(3) for λ_t and plugging into Eq.(4), get: (see Appendix 2)

$$(\frac{C_{t+1}}{C_t})^{\theta} = \beta B\{\frac{Y_{t+1}}{Y_t}\frac{H_t}{H_{t+1}}\frac{\phi_t}{\phi_{t+1}}\} \tag{6}$$

- Eq.(5) can be written as:

$$(\frac{C_{t+1}}{C_t})^{\theta} = \beta[\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)] \tag{7}$$

- Solution to the model is characterised by Eq.(1), (2), (6) and (7).

Balanced Growth Path (1 of 3)

- Finding growth rate: start from Eq.(2)

$$\frac{H_{t+1}}{H_t} = B(1 - \phi_t)$$

- Hence,

$$g = \frac{H_{t+1} - H_t}{H_t} = B(1 - \phi_t) - 1 \tag{8}$$

Balanced Growth Path (2 of 3)

- **Question**: Is there a balanced growth equilibrium where C_t, K_t, Y_t and H_t all grow at an identical constant rate while ϕ_t does not grow?
  - Look at Eq.(6) when C, Y and H all grow at an identical rate g, while ϕ does not grow, we have:

$$(1+g)^{\theta} = \beta B\{(1+g)\frac{1}{1+g}\frac{\phi}{\phi}\}$$

  - Therefore, we can find g

$$g = (\beta B)^{\frac{1}{\theta}} - 1$$

  - From Eq.(8), we also have $g = B(1-\phi^{*}) - 1$, we therefore can solve for ϕ^{*}.

Balanced Growth Path (3 of 3)

- The answer to the question is yes.
- They all grow at the same rate as human capital accumulation, $g = B(1-\phi^*) - 1 = (\beta B)^{\frac{1}{\theta}} - 1$, where $\phi^* = 1 - \frac{(\beta B)^{\frac{1}{\theta}}}{B}$ is the optimal allocation of individual's time between production and education.
- So that we get positive endogenous growth provided $\beta B > 1$. To get this to work, the returns to educations B have to be sufficiently high.

Remark on Lucas Model

- The Lucas model is elegant, but as always, it comes at expense of some realism.
- As Solow (1994) points out, the model's result is very sensitive to the assumption of constant return to capital (there is CRS in the human capital production) is problematic: a small deviation from this assumption would lead to either un-sustainable growth or explosive growth.
- The human capital accumulation equation assumes that the return to education is a constant, which is at odds both with the empirical evidence on education and with theories of human capital.

Technology Transfer with Education

- We argued in Lecture 7 that the difficulty poor countries face is not lack of access to advanced technology, but lack of ability to adopt the technology or apply it effectively.
- Among the various factors that may constrain a country's effective use of new technology, a critical factor is the level of educational attainment of the workforce.
- The theoretical model of Howitt and Mayer-Foulkes (2002) explicitly models the important role of skilled worker in the transfer of technologies.
- The empirical work of Griffith, Redding and Van Reenen (2001) find that education is an important determinant of the rate at which an industry in an OECD country can catch up to the world technology leader in that industry.

Final Comments on Growth Theory (1 of 3)

- The central question in growth theory is to account for the vast variations in GDP per capita over time (growth) and across countries.
- The Solow-Swan model predicts that if the market return to capital reflects its contribution to output, then variations in physical capital accumulation do not account for significant part of either worldwide economic growth or cross-country income differences.
- The R&D based endogenous growth models focus on knowledge accumulation as the primary driving force of growth, which seems promising for understanding worldwide growth while less convincing in accounting for cross-country income differences.

Final Comments on Growth Theory (2 of 3)

- Growth model with human capital and empirical growth accounting with human capital suggest that education plays an important role in accounting for income differences among countries.
- As we have seen, even though growth is an endogenous outcome in the endogenous growth models, its manifestation ultimately hinges on technology assumption that specify how technology or human capital enters production function and how it accumulates over time. It remains unclear how growth actually happens. In this sense, "endogenous growth" is not so "endogenous".
- Recent attempts in growth theory aim to go deeper and investigate the determinants underlying physical capital accumulation, human capital accumulation, technological progress and other factors directly related to economic growth.

Final Comments on Growth Theory (3 of 3)

- A leading candidate determinant is social infrastructure, by this, we mean institutions and policies that align private and social returns to activities. For example, legal systems, financial system, educational or health facilities, etc.
- Empirically, cross-sectional growth regressions have identified a positive relationship between growth rates and a variety of variables that measure developments in social infrastructure.
- A direction to push the endogenous growth literature forward is to incorporate institutional arrangements and/or government policies into growth models and quantitatively evaluate its significance in accounting for variations in per capita GDP.

Review of Original Kaldor Facts (1 of 3)

- Original Kaldor stylised facts summarise the patterns that economists had discovered in national income accounts.
- Shape the growth models that they were developing.
- The facts are:
  1. Labour productivity has grown at a sustained rate.
  2. Capital per worker has also grown at a sustained rate.
  3. The real interest rate or return on capital has been stable.
  4. The ratio of capital to output has also been stable.
  5. Capital and labour have captured stable shares of national income.
  6. Among the fast growing countries of the world, there is an appreciable variation in the rate of growth "of the order of 2 to 5 percent".

Review of Original Kaldor Facts (2 of 3)

- Initial neoclassical model of growth: Solow (1956) and Swan (1956), OLG and RCK framework.
- Great accomplishments of this theory.
  - It produced a single model that captured the first five of Kaldor's facts.
  - Explicit microeconomics foundations.
- Just one state variable : capital
- Faster growth come from higher rate of technological progress g_A (technological progress accounts for most growth).
- But g_A is exogenous in this model.
- Birth of endogenous growth theory.

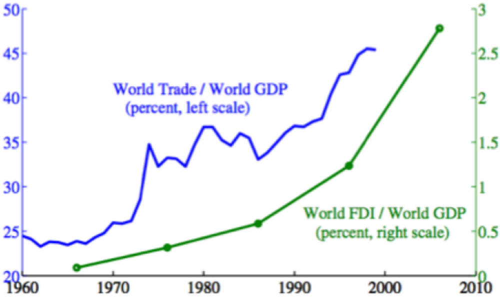
Review of Original Kaldor Facts (3 of 3)

- New growth models
  - Tent on making technological progress an endogenous part of a more complete model of growth.
  - Also brought into their models the other endogenous state variables excluded from consideration by the initial neoclassical setup.
    - Ideas, institutions, population, and human capital are now at the center of growth theory.
    - Physical capital has been pushed to the periphery.
  - So far growth models have been successful in capturing the endogenous accumulation of (and interaction between three of) four state variables: ideas, institutions, population, and human capital.
- To capture new facts, a growth model must consider the interactions between ideas, institutions, population, and human capital.
- What are these facts?

New Kaldor Fact 1: Increases in the Extent of the Market

- Increased flows of goods, ideas, finance, and people – via globalisation as well as urbanisation – have increased the extent of the market for all workers and consumers.

Figure: The rise in globalisation

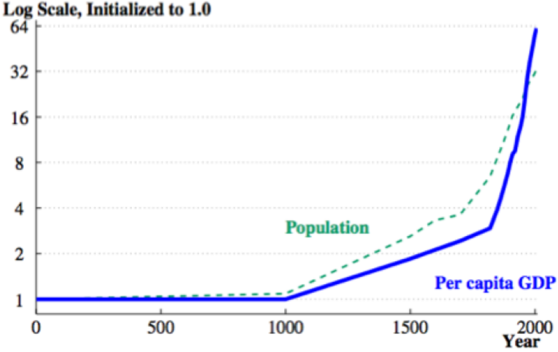


Note: World trade is the sum of world exports and imports as a share of world GDP from the Penn World Tables 6.1. FDI as a share of GDP is from the World Bank's *World Development Indicators*.

New Kaldor Fact 2: Accelerating Growth

- For thousands of years, growth in both population and per capita GDP has accelerated, rising from virtually zero to the relatively rapid rates observed in the last century.

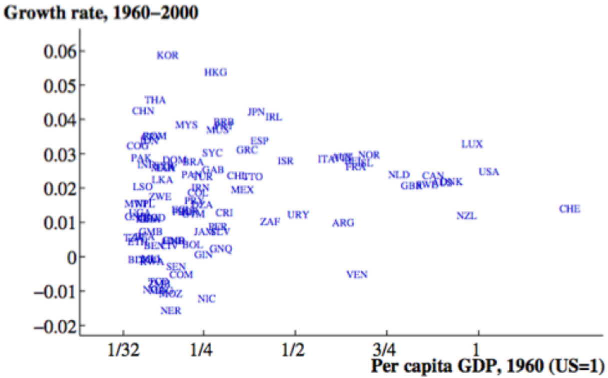
Figure: Population and per capita GDP in the very long run



New Kaldor Fact 3: Variation in Modern Growth Rates

- The variation in the rate of growth of per capita GDP increases with the distance from the technology frontier.

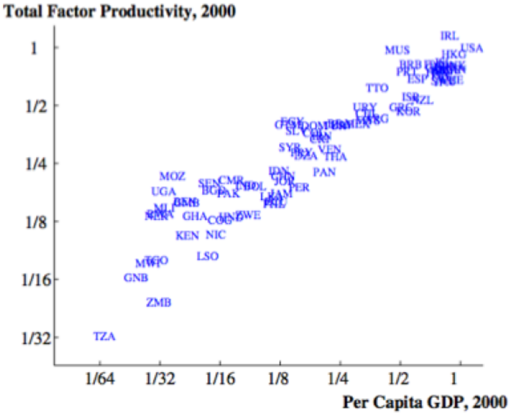
Figure: Growth variation and distance from the technology frontier



New Kaldor Fact 4: Large Income and TFP Differences

- Differences in measured inputs explain less than half of the enormous cross country differences in per capita GDP.

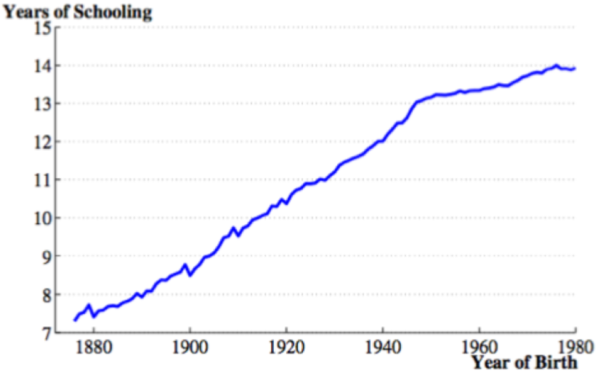
Figure: Large income and TFP differences



New Kaldor Fact 5: Increases in Human Capital per worker

- Human capital per worker is rising dramatically throughout the world.

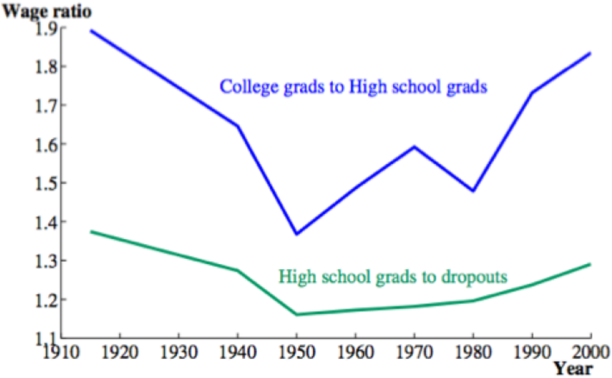
Figure: Years of schooling by birth cohort, United States



New Kaldor Fact 6: Long-run Stability of Relative Wages

- The rising quantity of human capital relative to unskilled labour has not been matched by a sustained decline in its relative price.

Figure: The U.S. college and high school wage premiums



Modern Growth Theory and New Kaldor Facts

- Fact 1 and 2: defining characteristic of ideas and their nonrivalry
  - The extraordinary rise in the extent of the market associated with globalisation and the acceleration over the very long run.
- Fact 3 and 4 : importance of institutions and institutional change
  - Enormous income and TFP differences across countries and the stunning variation in growth rates for countries far behind the technology frontier.
- Fact 5 and 6: emphasis is on human capital
  - Emphasis was on physical capital in the Kaldor's original observations.
- These facts also reveal important complementarities among the key endogenous variables.

Appendix 1 (1 of 2)

- Extend the utility. Recall that

$$C_t = K_t^{\alpha}(\phi_t H_t)^{1-\alpha} + (1-\delta)K_t - K_{t+1}$$

- We can then write the Lagrangian equation as

$$\mathcal{L} = \ldots + \beta^t \{ \frac{[K_t^{\alpha}(\phi_t H_t)^{1-\alpha} + (1-\delta)K_t - K_{t+1}]^{1-\theta} - 1}{1-\theta} + \lambda_t [B(1-\phi_t)H_t - H_{t+1}]\}$$
$$+ \beta^{t+1} \{ \frac{[K_{t+1}^{\alpha}(\phi_{t+1} H_{t+1})^{1-\alpha} + (1-\delta)K_{t+1} - K_{t+2}]^{1-\theta} - 1}{1-\theta} + \lambda_{t+1} [B(1-\phi_{t+1})H_{t+1} - H_{t+2}]\} \ldots$$

Appendix 1 (2 of 2)

- FOC w.r.t ϕ_t:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \phi_t} &= \beta^t \{ C^{-\theta} [(1-\alpha) K_t^{\alpha} \phi_t^{-\alpha} H_t^{1-\alpha}] + \lambda_t(-BH_t) \} \\
&= \beta^t \{ C^{-\theta} [\frac{\phi_t}{\phi_t} (1-\alpha) K_t^{\alpha} \phi_t^{-\alpha} H_t^{1-\alpha}] - \lambda_t B H_t \} \\
&= \beta^t \{ C^{-\theta} [\frac{(1-\alpha) K_t^{\alpha} \phi_t^{1-\alpha} H_t^{1-\alpha}}{\phi_t}] - \lambda_t B H_t \} \\
&= \beta^t \{ C^{-\theta} [\frac{(1-\alpha) Y_t}{\phi_t}] - \lambda_t B H_t \}
\end{aligned}
$$

- Follow this approach for the other two FOCs (they will be tested in problem set 2).

Appendix 2

- From Eq.(3), we find λ_t

$$\lambda_t = \frac{C_t^{-\theta}[\frac{(1-\alpha)Y_t}{\phi_t}]}{BH_t}$$

- Substitute the above equation into Eq.(4), get:

$$\begin{aligned}
\frac{C_t^{-\theta}[\frac{(1-\alpha)Y_t}{\phi_t}]}{BH_t} &= \beta\{C_{t+1}^{-\theta}[\frac{(1-\alpha)Y_{t+1}}{H_{t+1}}] + \frac{C_{t+1}^{-\theta}[\frac{(1-\alpha)Y_{t+1}}{\phi_{t+1}}]}{BH_{t+1}}B(1-\phi_{t+1})\} \\
\frac{C_t^{-\theta}Y_t}{BH_t\phi_t} &= \beta C_{t+1}^{-\theta}\{\frac{Y_{t+1}}{H_{t+1}} + \frac{Y_{t+1}(1-\phi_{t+1})}{H_{t+1}\phi_{t+1}}\} \\
\frac{C_t^{-\theta}}{C_{t+1}^{-\theta}} &= \beta B\frac{Y_{t+1}H_t}{Y_tH_{t+1}}\phi_t\{1 + \frac{(1-\phi_{t+1})}{\phi_{t+1}}\} \\
\frac{C_t^{-\theta}}{C_{t+1}^{-\theta}} &= \beta B\{\frac{Y_{t+1}}{Y_t}\frac{H_t}{H_{t+1}}\frac{\phi_t}{\phi_{t+1}}\}
\end{aligned}$$

Appendix 3: Empirical Growth Accounting with Human Capital (1 of 2)

- Theories imply that human capital could be an important factor in economic growth. Empirical growth accounting has tried to give effect to this insight in various ways, despite the obvious measurement difficulties.
- The U.S Bureau of Labour Statistics, in its own growth-accounting exercises, weights hours worked with relative wage rates such that more skilled workers (typically earn higher wages) get higher weights in calculating total productive labour services.
- Another way to measure human capital is to relate it to years of schooling. For example, see Hall and Jones (1999) and Klenow and Rodriguez-Clare (1997), where a growth accounting exercise is implemented across countries to determine to what extent income differences among countries are due to differences in physical capital accumulation, differences in human capital accumulation, and other factors.

Appendix 3: Empirical Growth Accounting with Human Capital (2 of 2)

- In these studies the stock of labour services i in country is measured as

$$H_i = e^{\phi(x_i)} L_i$$

  where x_i is the average number of years of education of workers in country i, ϕ is an increasing function which they can estimate using micro data on earning and years of schooling of employees.
- They find that about $1/6$ of the gap between per capita income in richest and poorest countries is due to differences in physical-capital intensity, less than $1/4$ is due to all other factors (not necessarily technology).
- Their work has been extended in numerous ways in the empirical literature. For the most part, the extensions suggest an even larger role for the residual.